Distribution network management based on optimal power flow: integration of discrete decision variables

Enrique Briglia, Sebastián Alaggia and Fernando Paganini
Universidad ORT Uruguay

***Abstract*—Recent developments in the optimal power flow (OPF) problem for radial networks open the promise of a more sophisticated management of the distribution end of the power grid. Such sophistication is required to operate efficiently new, dynamic energy resources being introduced at the micro scale. To be useful, however, optimization tools must also accommodate pre-existing management technologies which involve discrete decisions. In particular for topology reconfiguration, we extend recent references to provide a new, exact characterization of admissible topologies, explaining in detail its integration with OPF. We also propose methods to include discrete decisions on transformer taps and capacitor banks, as well as ON/OFF loads, to yield mixed-integer nonlinear programs with convex integer relaxations. We report simulation results on the application of this methodology to a distribution network in Uruguay.**

I. Introduction

Significant changes are under way in the power grid, with the introduction of renewable energy sources [9], demand-side management [10], and new possibilities for electricity storage [3]. Many of these technologies will enter the grid in a disseminated manner, at the level of the *distribution* network. This prospect alters the traditional picture of a grid where energy is generated in large-scale facilities connected at the transmission level, and conveyed down to loads through a passive, mostly static distribution network.

Endowing the distribution network with distributed generation (DG), storage (DS), and demand response (DR) poses significant operational challenges. In contrast to the transmission grid, lower voltage sections are usually not overprovisioned in capacity; this implies that power flows varying dynamically in an uncoordinated manner cannot always be accommodated within operational limits, for instance in network voltages. A network-aware dispatch of these distributed units is required, integrated with the power flow (PF) equations that determine the electrical variables; running PF offline for a few hand-picked scenarios will not suffice. Rising to the challenge are recent advances in the Optimal Power Flow (OPF) [11], which rely on convex relaxations to provide reliable solutions, and could form the basis of an operation and dispatch tool for future distribution networks.

The OPF methodology has, however, still a road to travel to be adopted by distribution grid operators (DGOs). In particular, DGOs will want an environment in which the traditional methods employed for operation are included along with the new. These older methods involve discrete decision variables, as in the reconfiguration of topology through switches, voltage adjustments by transformer taps, capacitor banks, etc., which go out of the realm of convex optimization. Discrete variables also appear in the ON/OFF dispatch decisions of early adopters of demand response, such as large industrial processes.

Another important requirement is that the OPF methodology should reflect, as faithfully as possible, the real operational costs faced by the DGO. In previous work [2] we provided such a cost function, modeling real conditions of the Uruguayan grid. Convexified OPF models of the network as in [4], together with detailed models of wind and solar DG equipment, were considered. The discrete decision variables were, however, only dealt with in [2] through exhaustive search, which limited the achievable problem size.

In this paper we expand the approach to optimize over the discrete decision variables in a systematic way, taking advantage of new mixed-integer programming tools (e.g. [13]) tailored to problems with a convex integer relaxation. A method of this kind combining OPF with topology reconfiguration variables was proposed in [7], [16], and shown to be correct for passive distribution networks, i.e. with power flowing down from substations. In Section II we extend this approach by proving that a set of linear mixed integer conditions exactly characterizes the admissible topologies (irrespective of the direction of power flow), and enables the joint optimization with OPF. Methods to include transformer taps, capacitor banks, and ON/OFF loads are also presented.

Section III presents an application of the above methodology to optimize a 100 bus network obtained by incorporating DG and other discrete degrees of freedom to a real feeder in Uruguay. The implementation in Matlab/CVX [5] with the Mosek [13] solver obtains interesting results in moderate amounts of computation time. Conclusions and future extensions are outlined in Section IV.

II. Discrete operation decisions with convex OPF

The last few years have seen remarkable advances in the classical (AC) OPF problem: through appropriate relaxation, convex optimization problems have been proposed [11] for the efficient computation of global bounds, which are sometimes exact. Of particular relevance here are the convex relaxations [4] of the DistFlow [1] model, which tend to give exact solutions in radial networks. OPF involves *continuous* variables (active and reactive power flows, voltages, currents), jointly optimizing over the available degrees of freedom (e.g. reactive power in inverters of DG sources [4]).

However there are also, traditionally, *discrete* decision variables in the hands of the DGO: reconfiguration switches, transformer taps, etc. Adding these as optimization variables breaks convexity and would seem to compromise all recent advances in OPF. Nevertheless, mixed-integer programs can remain tractable as long as their continuous relaxations are convex. The rationale is that in a branch and bound procedure (see e.g. [6]) over the discrete variables, the bounding part uses such relaxations. While these are NP-hard problems so worst-case computation time is long, the procedure can achieve close to optimal configurations in moderate amounts of time, and furthermore the gaps to optimality can be bounded. We now pursue this approach for different discrete decisions.

A. Characterization of Admissible Network Topology

In the traditional view, power flows down from the transmission grid to a set of distribution feeders, arranged at any given time in a radial topology. This set of tree networks may, however, be reconfigured through switches, for instance transferring load from one substation to another. This provides backup to address contingencies, and also has impact on voltage profiles which may be exploited for quality assurance.

In future grids endowed with DG or DS, the concept of *feeder* is itself in question, since power may flow in different directions; still, there are good reasons for distribution networks to continue to operate radially and with connectivity to the transmission grid. On one hand, allowing parallel branches in the topology (as commonly used in transmission) is not an attractive option for the heterogenous, not heavily-invested distribution networks: indeed, the division of flows between paths is highly dependent on the quality of lines and connectors, whose uniformity is difficult to guarantee. Furthermore, allowing a portion of the network to operate as an island, disconnected from transmission, is undesirable both for reasons of robustness of power supply, and the fact that most DG sources connected through inverters are devised to obtain their AC synchronism from the grid. This leads us to state the following topology requirement:

Definition 1. *A distribution network graph is* admissible *if it is a union of disjoint trees, each of which contains exactly one substation*[1].

The problem of optimizing over such configurations in conjunction with the continuous PF variables, has been pursued recently in [7], [14], [16]. In particular [16] devised a scheme involving a binary variable y_{ij} for each line with a switch, plus two orientation variables z_{ij}, z_{ji} for every line, optimized jointly with different approximations to the DistFlow [1] model. More recently, [7] extended this approach to the convex (SOCP) relaxation of DistFlow, as in [4].

While the (fewer) switch variables must be treated as discrete, it is established in [16] that under suitable linear restrictions (see (2a-2d) below) the solutions for $z_{ij}, z_{ji} \in [0,1]$ are integer; they are used to specify an oriented graph which is admissible. However there is one step in their argument (establishing the absence of loops) which assumes a *passive* network, incapable of achieving power balance other than by drawing from the grid. In a more general context where DG or DS is present, this argument breaks down. In what follows we extend the method of [16] to impose admissibility of the graph, *independently* of any power considerations.

Our distribution network is defined by an undirected graph $\mathcal{G} = (\mathcal{N}, \mathcal{L})$ with $|\mathcal{N}| = N$ nodes ("buses") and $|\mathcal{L}| = L$ edges ("lines"). A distinguished set of nodes $\mathcal{S} \subset \mathcal{N}$ are termed "substations". There are no direct lines between substations (the transmission network is external to our graph).

[1] If desired, a reliable synchronous DG could be treated as a substation.

A subset $\mathcal{L}_{sw} \subset \mathcal{L}$ of lines contain *switches*. For each such line define a binary decision variable

$$y_{ij} \in \{0,1\}, \tag{1}$$

where $y_{ij} = 1$ indicates the switch is closed. Out of the $2^{|\mathcal{L}_{sw}|}$ possible combinations of the decision variables, we wish to characterize those which result in an *admissible* network. For there to be any such solutions a necessary condition is:

Assumption 1. *When all switches are open, the network graph contains no loops (cycles).*

Define now the following auxiliary variables:

- $z_{ij}, z_{ji} \in [0,1]$ for each line $\{i,j\} \in \mathcal{L}$;
- a "potential" $W_i \in [0, N-1]$ for each node $i \in \mathcal{N}$,

and impose the following restrictions:

$$z_{ji} = 0, \quad i \in \mathcal{S}; \tag{2a}$$
$$\sum_{j:\{i,j\}\in\mathcal{L}} z_{ji} = 1, \quad i \in \mathcal{N}\backslash\mathcal{S}; \tag{2b}$$
$$z_{ij} + z_{ji} = 1, \quad \{i,j\} \in \mathcal{L}\backslash\mathcal{L}_{sw}; \tag{2c}$$
$$z_{ij} + z_{ji} = y_{ij}, \quad \{i,j\} \in \mathcal{L}_{sw}; \tag{2d}$$
$$z_{ij} \in \{0,1\}, \quad \{i,j\} \in \mathcal{L}_{sw}; \tag{2e}$$
$$W_j \geq W_i + 1 - N(1 - z_{ij}), \quad \forall\ \{i,j\} \in \mathcal{L}. \tag{2f}$$

Remark 1. *Constraints (2a-2d) are identical to those in [16]; we have only added (2f), and imposed an integrality constraint (2e) on the z variables for lines* with switches. *This latter change adds computational complexity to the situation of [16], where only the y_{ij} were forced to be binary; but since the vast majority of lines are not in this class, the impact is moderate. Other than this fact, the constraints in (2) are* linear.

Given a choice for the decision variables y_{ij}, $\{i,j\} \in \mathcal{L}_{sw}$, let $\mathcal{G}'$ be the graph obtained from $\mathcal{G}$ by eliminating the branches with $y_{ij} = 0$.

Theorem 1. *The following are equivalent:*

(a) *$\mathcal{G}'$ is an admissible network.*

(b) *There exist $W_i \in [0, N-1]$, $z_{ij} \in [0,1]$, $z_{ji} \in [0,1]$, satisfying (2).*

Furthermore, under these conditions any solution to (b) must satisfy $z_{ij} \in \{0,1\}$ for every *$\{i,j\} \in \mathcal{L}$.*

Proof: To establish that (a) implies (b), orient each tree in $\mathcal{G}'$, outward with root at the substation. Assign $z_{ij} = 1$, $z_{ji} = 0$ when i is the parent of j in this tree. Also set z variables for open switches to zero. This assignment meets (2a)-(2e). Also assign $W_i = 0$ to each substation, $W_j = 1$ for each child, $W_k = 2$ for the second generation, etc. The potentials stay smaller than the bound $N-1$. By construction (2f) holds when j is the child of i ($z_{ij} = 1$), but also when $z_{ij} = 0$ since we always have

$$W_j \geq 0 \geq W_i - (N-1).$$

We now show that (b) implies (a). In the process we also show that all the z variables in the solution must be integer.

Consider, in the reduced graph $\mathcal{G}'$, the set $\mathcal{N}_i$ of nodes connected to substation i; it could be the singleton $\{i\}$, a trivial case of a tree as required for (a). Otherwise, let j be a neighbor of i, we have $z_{ji} = 0$ from (2a) and thus[2] $z_{ij} = 1$; we label such neighbors as "children" of i in a directed graph with orientation given by z_{ij}. Now consider, for the child node j, another neighbor $k \neq i$; since $z_{ij} = 1$, (2b) implies that $z_{kj} = 0$ for any such k. Therefore we must have $z_{jk} = 1$ and these nodes k become children of j. In particular note that k cannot be another substation due to (2a), or another child of the first generation, because by (2b) the parent is unique. Continuing inductively, further generations of children are added until reaching stub nodes that have no further neighbors. So $\mathcal{N}_i$ is a tree rooted in i, with no other substations, and $z_{ij}, z_{ji} \in \{0, 1\}$ for all its links.

Repeating the procedure for every substation gives a disjoint family of trees. It remains to show that they cover all of $\mathcal{G}'$. Assume instead there is a connected component $\mathcal{G}'' = (\mathcal{N}'', \mathcal{L}'')$ within $\mathcal{G}'$, that does not include a substation. The following identity must hold due to (2c-2d) and (2b):

$$|\mathcal{L}''| = \sum_{\{i,j\} \in \mathcal{L}''} (z_{ij} + z_{ji}) = \sum_{i \in \mathcal{N}''} \left(\sum_{j:\{i,j\} \in \mathcal{L}''} z_{ji} \right) = |\mathcal{N}''|;$$

in words, each link contributes a total of 1 unit of "z-flow" entering its end nodes; since each node can receive 1 unit total, the number of links and nodes must match.

Since a spanning tree inside $\mathcal{G}''$ has $|\mathcal{N}''| - 1$ links, there is only one more link in $\mathcal{G}''$. Adding one link to a tree gives a graph with a *single* cycle. There may be trees spawning out of the cycle, but these contribute no injection of "z-flow" to the nodes in the cycle. To see this, start with a stub node j in such a tree, by (2b) it must receive $z_{ij} = 1$ from its parent i and return $z_{ji} = 0$; so removing this link does not affect the z-flow balance of the parent. This removal can be repeated recursively until one is only left with the cycle.

Now, by Assumption 1 the cycle must contain at least one line with a (closed) switch, $\{i, j\} \in \mathcal{L}_{sw}$. By hypothesis, the corresponding z_{ij} is *binary*; assume for instance $z_{ij} = 1$, $z_{ji} = 0$. Now by (2b) the next neighbor k of j along the cycle must satisfy $z_{kj} = 0$, hence $z_{jk} = 1$, and this argument repeats until the cycle is closed. But this contradicts (2f): if $z = 1$ along a path the potential must strictly increase, so it cannot close. This implies $\mathcal{G}''$ must be empty, as claimed. ■

The preceding theorem provides an exact characterization of the degrees of freedom available in the network topology, to be optimized jointly with others for an efficient operation. It is independent of any electrical or power considerations; it does not rely on the passivity of the network, and there is *no* need for $z_{ij} = 1$ to match the direction of power flow.

Example 1. *To motivate the integrality requirement (2e), consider the toy network of Fig. 1. A simple analysis reveals that the admissible configurations are those that close exactly one of the switches 1-2 or 3-5, and exactly one of 2-4 or 3-4.*

[2]By (2c-2d), $z_{ij} + z_{ji} = 1$ holds for all lines in $\mathcal{G}'$.

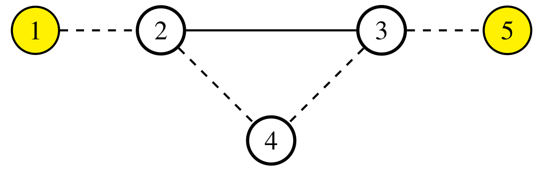

Fig. 1: Example network. Substations are nodes 1 and 5. Dashed lines have switches.

However, consider the possibility of closing only switches 2-4 and 3-4, which is not admissible since it creates a cycle disconnected from transmission. The choice of variables

$$z_{12} = z_{21} = z_{35} = z_{53} = 0,$$
$$z_{23} = z_{32} = z_{24} = z_{42} = z_{34} = z_{43} = \frac{1}{2},$$

together with $W_i = 0$ for all i verifies (2a-2d) and (2f). So we cannot rule out this possibility without the constraint (2e).

The preceding example says that if we want a priori guarantees that the solution z will be binary, *a few* such constraints must be included; Theorem 1 proves it suffices to include those for switched lines. Of course, it may well be that running our OPF (discussed further below) with only the linear constraints for z returns an integer solution; in that case we are spared of the computational burden of (2e). If it fails, we can re-run with this additional constraint.

B. Integration with OPF

To integrate the topology variables with the power flow equations, we will follow for the most part the approach in [7], with minor variations; however we contribute what is in our view a more precise and self-contained justification.

The PF model is of the *branch flow* kind, see [11] for extensive background. Such models pick an (arbitrary) orientation for each line, and define *one* set of variables P_{ij}, Q_{ij} per line, representing *sending side* real and reactive powers, from i to j according to this orientation. For radial networks a common choice is to orient the graph downstream from the (substation) root. Note that this does *not* imply power flows downstream: P_{ij}, Q_{ij} could be negative in a network with DG or DS.

We would like to follow a similar procedure, except here the downstream direction is not available a priori, it depends on the topological decision variables. For instance in Fig. 1, if switch 1-2 is closed then node 3 is downstream of node 2, but the opposite happens if 3-5 is closed. The solution is to introduce *both* P_{ij}, P_{ji} variables for each line (and similarly Q_{ij}, Q_{ji}) but let the graph orientation variables z_{ij}, z_{ji} in (2), force one of each pair to be zero. Similarly we will allow a priori two variables l_{ij}, l_{ji}, one of which will represent the magnitude square of line current, the other one will be zero. Write the following conditions for each node j:

$$\sum_{i:\{i,j\} \in \mathcal{L}} (P_{ij} - r_{ij} l_{ij}) = p_j + \sum_{i:\{i,j\} \in \mathcal{L}} P_{ji}; \quad \text{(3a)}$$

$$\sum_{i:\{i,j\} \in \mathcal{L}} (Q_{ij} - x_{ij} l_{ij}) = q_j + \sum_{i:\{i,j\} \in \mathcal{L}} Q_{ji}, \quad \text{(3b)}$$

together with the following branch constraints for each $\{i, j\}$:

$$P_{ij}^2 + Q_{ij}^2 \le v_i l_{ij}, \tag{4a}$$
$$l_{ij} \le z_{ij} \bar{l}_{ij}. \tag{4b}$$

Here: p_j, q_j are real and reactive powers extracted from node j by external units (we chose the positive sign for loads), v_i the magnitude square of the bus voltage. r_{ij}, x_{ij} are line resistance and reactance, and $\bar{l}_{ij}$ a current line limit.

On its own, (3) is *not* a valid power balance equation. For it to become one, each line should appear only one side, right or left, of the equation. However, *given* a set of z variables under (2), one of z_{ij}, z_{ji} is zero for each closed line, forcing through (4) the corresponding l, P and Q variables to zero, and making the "accounting" in (3) meaningful in the remaining variables. In fact, since by (2b) there is only one parent node i (with $z_{ij} = 1$) for each $j \in \mathcal{N} \backslash \mathcal{S}$, (3a) will become equivalent[3] to:

$$P_{ij} - r_{ij} l_{ij} = p_j + \sum_{k: z_{jk}=1} P_{jk};$$

this is the standard branch flow equation for active power (eqn (18) in [4]), for the orientation defined by z; an analogous one holds for reactive powers. Condition (4a) is the conic relaxation of the identity that relates currents, voltages and powers ((21) in [4]).

The voltage drop per line is imposed as in [7]:

$$v_j \ge v_i - 2(r_{ij} P_{ij} + x_{ij} Q_{ij}) + (r_{ij}^2 + x_{ij}^2) l_{ij} + (\underline{v} - \bar{v})(1 - z_{ij}); \tag{5a}$$
$$v_j \le v_i - 2(r_{ij} P_{ij} + x_{ij} Q_{ij}) + (r_{ij}^2 + x_{ij}^2) l_{ij} + (\bar{v} - \underline{v})(1 - z_{ij}), \tag{5b}$$

where $\underline{v} < 1 < \bar{v}$ and we also assume

$$\underline{v} \le v_i \le \bar{v}, \quad i \in \mathcal{N} \backslash \mathcal{S}; \tag{6a}$$
$$v_i = 1, \quad i \in \mathcal{S}. \tag{6b}$$

In [7] the expressions (5) are explained as convex relaxations from *disjunctive programming*. A disjunctive constraint is a union of two convex alternatives, and its convex hull is parameterized by a *non-integer* variable z_{ij}, subject to a number of constraints, two of which would be (5); [7] suggests the relaxation could be strengthened with more constraints.

This motivation is unclear to us since as explained, (3) is only meaningful under *binary* z_{ij}; if this is guaranteed using Theorem 1, (5) can be more easily explained and allows no strengthening. Indeed, whenever $z_{ij} = 1$, (5) amounts to a single equality, which is the standard Distflow voltage drop for the line (see (20) in [4]); when $z_{ij} = 0$, as argued before P_{ij}, Q_{ij}, l_{ij} are all zero and (5) holds trivially from the voltage bounds (6a), so the restriction disappears as desired.

C. Transformer taps

A standard method for voltage control in distribution networks is by means of transformers, adjusting through taps their transformation ratio. In per-unit terms, from the default of unity the transformation ratio would have the form $1 + \tau N^{tr}$, where τ is a tap step (e.g. 0.025) and N^{tr} is an integer around zero, e.g. $N^{tr} \in \{-5, \dots, 0, \dots, 5\}$. To include such transformer in a certain line implies replacing v_i on the right-hand side of (5) by a new variable v_i^n, which must satisfy

$$v_i^n = n_i v_i = (1 + \tau_i N_i^{tr})^2 v_i, \tag{7}$$

for some N_i^{tr} in the integer range[4]. As mentioned before, the integer constraint may be accommodated by branch and bound solvers, *provided* its relaxation is convex; this unfortunately is not the case with (7). There are two issues: one, the multiplication $n_i v_i$; second, the quadratic expression for n_i, which appears since v_i is the *square* of voltage magnitude.

If the transformer is at a substation bus, the first difficulty disappears due to (6b); focusing first on this case, consider convex approximations to the quadratic. The simplest is the linearization $n_i = (1 + \tau_i N_i^{tr})^2 \approx 1 + 2\tau_i N_i^{tr}$ around $N_i^{tr} = 0$, which may be quite accurate but does not offer guarantees since it does not "cover" the nonlinearity. To achieve the latter we can write the McCormick [12]-type convex inequalities

$$n_i \ge (1 + \tau_i N_i^{tr})^2; \tag{8a}$$
$$n_i \le 1 + 2\tau_i N_i^{tr} + \tau_i^2 (\underline{N}_i^{tr} N_i^{tr} + \overline{N}_i^{tr} N_i^{tr} - \underline{N}_i^{tr} \overline{N}_i^{tr}), \tag{8b}$$

valid in $N_i^{tr} \in [\underline{N}_i^{tr}, \overline{N}_i^{tr}]$. However, an undesirable feature of this relaxation is that at the nominal point $N_i^{tr} = 0$, the upper bound (8b) is off by a non-trivial amount. An alternative is to use a piecewise linear upper bound in the intervals $[\underline{N}_i^{tr}, 0]$, $[0, \overline{N}_i^{tr}]$, replacing (8b) by

$$\begin{aligned} n_i &\le 1 + 2\tau_i N_i^{tr} + \tau_i^2 \underline{N}_i^{tr} N_i^{tr} + M b_i; \\ N_i^{tr} &\le M b_i; \\ n_i &\le 1 + 2\tau_i N_i^{tr} + \tau_i^2 \overline{N}_i^{tr} N_i^{tr} + M(1 - b_i), \\ N_i^{tr} &\ge -M(1 - b_i), \end{aligned} \tag{9}$$

where M is a "big" constant and $b_i \in \{0, 1\}$ is an additional binary variable selecting the subinterval.

If a transformer is placed in a bus with variable voltage, an additional relaxation is required to accommodate the product $n_i v_i$ in (7); a standard choice is the McCormick relaxation, analogous to the one presented below for capacitors.

D. Capacitor banks

Another standard tool for volt-var control is the installation at certain nodes of banks of capacitors, with a discrete choice for the amount of capacity to be connected at a given time. In the model, this amounts to a reactive power load $q_j^{cp} = -Cap_j N_j^{cp} v_j$ at node j, as (part of) the load in (3). Here Cap_j is the baseline unit of added capacity, and $N_j^{cp} \in \{\underline{N}_j^{cp} = 0, \dots \overline{N}_j^{cp}\}$ the integer multiple employed.

Relaxing the integer constraint leaves a multiplicative nonlinearity, which can be bounded in the McCormick fashion:

$$-q_j^{cp}/Cap_j \ge N_j^{cp} \underline{v} + \underline{N}_j^{cp} v_j - \underline{N}_j^{cp}\ \underline{v}; \tag{10a}$$
$$-q_j^{cp}/Cap_j \ge N_j^{cp} \bar{v} + \overline{N}_j^{cp} v_j - \overline{N}_j^{cp}\ \bar{v}; \tag{10b}$$
$$-q_j^{cp}/Cap_j \le N_j^{cp} \underline{v} + \overline{N}_j^{cp} v_j - \overline{N}_j^{cp}\ \underline{v}; \tag{10c}$$
$$-q_j^{cp}/Cap_j \le N_j^{cp} \bar{v} + \underline{N}_j^{cp} v_j - \underline{N}_j^{cp}\ \bar{v}. \tag{10d}$$

[3] They are not "identical" equations as stated in [7]; the equivalence only holds when combined with (4) under a given z satisfying (2).

[4] $\underline{v}, \bar{v}$ in (5) must also be replaced by bounds that hold for all N_i^{tr}.

E. ON/OFF industrial loads

Much of the literature on demand response (see e.g. [15]) considers a *continuous* adjustment of power consumption in response to pricing signals from the grid. There is, however, a more basic form of DR in industrial customers, which concerns the dispatch of an entire process, as such a *discrete* decision. Indeed, due to their relative sophistication these customers are likely to be sooner in a position to respond to economic signals.

Consider a discrete time horizon $t = 1, \dots, T$ (e.g. a day), and assume the process must operate in a non-interruptible way for a duration of D time slots. During its operation it has a fixed power consumption, but the owner incurs an additional cost $C^c(t)$ which may be time dependent (reflecting, e.g., variable labor costs). The question is how to parameterize the discrete decision of when to start the process, integrated into the OPF under the paradigm of mixed-integer optimization with convex relaxation. The solution involves two binary functions $st(t) \in \{0,1\}$ (indicator of start time) and $on(t) \in \{0,1\}$ (indicator of ON time) and the following linear constraints[5]:

$$\sum_t st(t) = 1, \quad \sum_t on(t) = D,$$
$$st(t) + on(t) \geq on(t+1), \quad t = 1, \dots, T-1. \qquad (11)$$

This imposes a single start time, a duration D, and non-interruptibility: indeed, from (11) the process can only be ON following another ON slot or the start signal. The consumption $p^n on(t)$ is then included in the corresponding bus; a reactive power consumption $q^n on(t)$ may also appear.

III. Optimization Example

Our testbed for distribution network management was introduced in [2]; here we apply to it a systematic treatment of discrete decision variables.

A. Cost function

The cost function attempts to reflect the real decision making environment of a DGO in Uruguay, over a time horizon $t = 1, \dots T$, in this case 24 hours. Its form is:

$$C = \sum_{t=1}^{T} \left[C^{tr}(t) + C^v(t) + C^f(t) + C^{om}(t-1,t) + C^c(t) \right],$$

where (for more details see [2]):

$C^{tr}(t) = \pi_p^{tr} p_0(t) + \pi_q^{tr} |q_0(t)|$ is the cost for purchased power at the transmission bus 0 (we allow selling active power as a good, reactive power is always penalized);

$C^v(t) = \sum_{j \in N} c_j^{v_j}(t)$ are regulator penalties for voltage operating outside the range $[1-\delta_v, 1+\delta_v]$ in p.u.; penalties are taken to be linear in v_j outside this range;

$C^f(t)$ are penalties for frequency of service interruption, which is a function of the topology and branch failure rates;

$C^{om}(t-1,t)$ are operation and maintenance costs associated with switching components (switches, taps, caps);

$C^c(t)$ is the consumer cost, for customers with DR. For loads as in Section II-E we have a time dependent cost for operating at a certain time of day.

[5]We acknowledge [8] for suggesting this formulation.

B. Case study

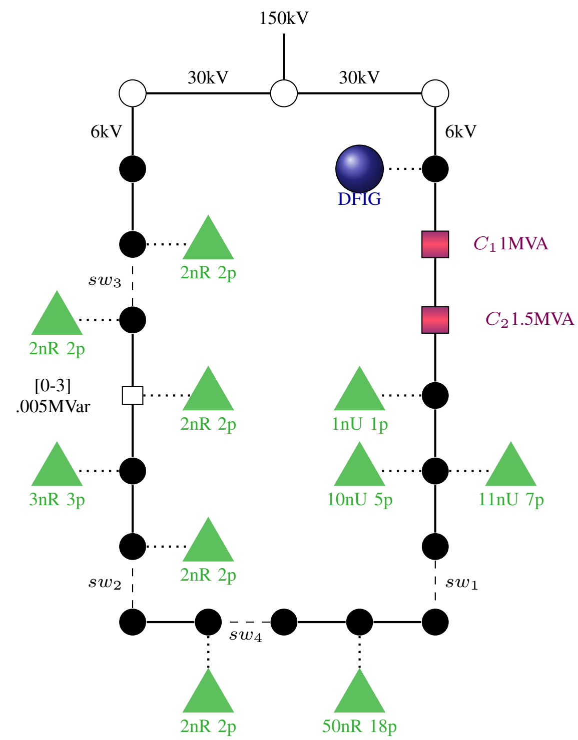

Fig. 2: Network example. Ring topology with 20 buses, plus another 85 buses in tree subnets fed from the main loop, indicated by triangles: label XnY Zp where X is # of buses, Y the type (rural or urban) and Z the tree depth. Squares indicate buses with industrial loads (right) and capacitor bank (left). Relevant switches are dashed lines.

The network represented in Fig. 2 is a real medium-voltage network in the suburban region of Montevideo, with some additions described below. The connection to Transmission is through a 150KV/30KV transformer with adjustable taps under load. The two transformers to 6KV, designed respectively for the rural and urban portions of the network, are not adjustable. The rural portion, on the left and bottom branches, is a long line supported on wooden posts, with high failure rate. The urban portion on the right corresponds to a subterranean cable. The low voltage buses are represented in a summarized way, indicating type (rural or urban) and tree depth. There are currently 3 main switches $sw1 - sw3$, the normal configuration being with $sw1$ open and the others closed. Changing these positions allows for the rural portions to be fed through the urban side, to handle contingencies in the less reliable lines.

To the existing network we have incorporated these additions: a 1.5 MVA Wind Generator, indicated by DFIG; two non-interruptible industrial loads of 1.5MVA and 1MVA with respective durations of 12hs and 8hs, and a capacitor bank. We also explore the inclusion of switches in *all* the branches of the medium voltage loop between the two feeders; the figure identifies one ($sw4$) which appears in the solution.

C. Results

The optimization was programmed in CVX over Matlab [5], and runs the mixed integer solver Mosek [13]. The cost function is as in Section III-A, and the constraints applied are (1-2) for the topology variables, (3-6) for OPF, (8a-9) for the transformer, (10) for the capacitor, and (11) for the industrial loads. We refer to [2] for the OPF model of the wind generator.

The resulting problem had $\sim 10^6$ variables of which $\sim 4{\cdot}10^4$ are integer, with $\sim 7{\cdot}10^5$ linear constraints and $\sim 4{\cdot}10^4$ cone constraints. We used an i5 PC, 3.6 GHz and 8GB RAM. Compilation time from CVX takes 15min, and the solve time was limited to 20 min. While there was still a 10% gap in the branch and bound at this run time, it already found interesting configurations.

We optimized over 24 hours under the following conditions: the wholesale price of active power increases by 30% at 7 PM. Wind speeds and loads are taken from historical records of the peak load day in 2013; wind exhibits a lull in the early PM, a typical situation. For the industrial loads, the preferred time range of operation is between 7 AM and 10PM, outside this range labor costs triple.

Two cases were considered, represented in Figs. 3 and 4. In the first, we only enabled the preexisting switches; the second explores for a better cut among all lines in the medium voltage loop. In both solutions, the industrial loads were placed at disjoint intervals of time, avoiding the low wind hours. The transmission transformer is operated at tap position $N_0 = 2$.

The solutions differ on the choice for switches and capacitors. When only $sw1 - sw3$ are available, the network opens $sw2$ and does not activate the Caps. If instead we search freely for a line to open, it opens $sw4$, and includes the Caps. We note that the line of the bottom left of Fig. 2, now fed through the rural feeder, is much longer than the rest; it thus makes sense to isolate its higher failure rate from the urban side; the capacitor assists with voltage control on the rural side.

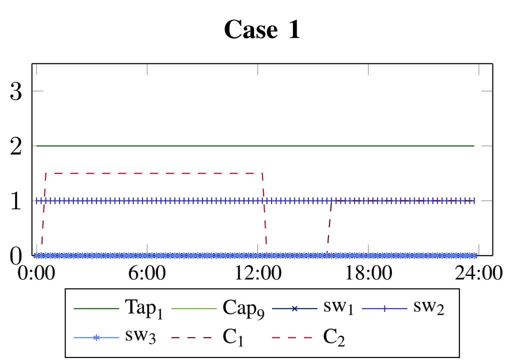

Fig. 3: Only switches $sw1, sw2, sw3$ enabled.

IV. Conclusions and Future work

We have presented methods to integrate discrete decisions into optimal power flow, with convex integer relaxations. State-of-the-art mixed integer solvers are then used to solve optimization problems at the scale of a distribution network. Preliminary experimental results look promising, but far more testing is required to test the limits of this procedure. We are also are working on integrating other forms of DG, DS and DR, and distributed implementations.

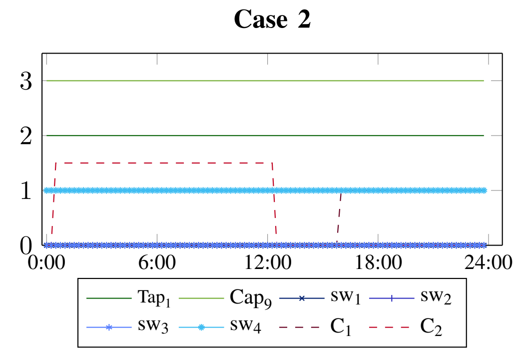

Fig. 4: Switches enabled in all 6KV lines.

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